Answer to the Comment about the Letter entitled "Scalar fields as dark matter in spiral galaxies"

F. Siddhartha Guzmán and Tonatiuh Matos

Departamento de Física

Centro de Investigación y de Estudios Avanzandos del IPN

Apdo. Postal 14-740 México 07000 D.F., México

February 7, 2008

The comment about reference [1] starts with false arguments and therefore the conclusions of the comment are wrong. Nevertheless, the arguments used by the author are often motive of confusions and it is worth to publish this answer in order to avoid misunderstandings in the future.

It is well known that there are no non-singular asymptotically flat, static solutions to the coupled Einstein Scalar Field theory. However, as it is discussed in [2], the space-time of a galaxy cannot be asymptotically flat, otherwise, the rotation curves of the galaxy would decay. Observations show that rotation curves in galaxies grow up for larger radii or remain flat, but in general they do not decay. The "source" of the scalar field should therefore provoke a non-asymptotically flat space-time as well. The space-time of the paper [1] is singular only for $r = 0$, but as it is specified in it, it pretends to be only the metric of the dark matter dominated region. The study of the center of the galaxy is much more complicated because we do not have any direct observation of it. If we follow the hypothesis of the scalar dark matter, we cannot expect that the center of a galaxy is made of "ordinary" matter, we expect that it contains baryons, self-interacting scalar fields, etc. There the density contrast of baryonic matter and scalar dark matter is the same (see [3]) and their states are in extreme conditions. As long as we know, there are no existence-uniqueness theorems for non-asymptotically flat, axial symmetric space-times under these conditions.

Furthermore, the energy conditions are no longer valid in nature, as it is shown by cosmological observations on the dark energy. Why should the energy conditions be valid in a so extreme state of matter like it is observed in the center of galaxies? If dark matter is of scalar nature, why should it fulfill the energy conditions in such extreme situation? As long as we know, there is no uniqueness theorem in the presence of matter like in this situation either.

The second objection of the author is related with the geodesics of the metric. In fact, this point is not clear enough in the paper [1], but its statement is not new, it was discussed in [4] and maybe it could be the reason to publish the comment. In [2] we found that if the dark matter is scalar, either the scalar potential vanishes or the velocities of the stars are luminal. This is equivalent to the statement of the author of the comment. However, even when the luminous matter only represents about 5-10% of the whole matter of the galaxy, it is crucial for its stability. The geodesic equations of the metric (21) of reference[1] (representing only the dark matter component) in the equatorial plane read

$$\frac{d^2D}{d\tau^2} - D\left(\frac{d\phi}{d\tau}\right)^2 + f_0^2c^2D\left(\frac{dt}{d\tau}\right)^2 = 0, \quad \frac{d\phi}{d\tau} = \frac{B}{D^2f_0}, \quad \frac{dt}{d\tau} = \frac{A}{c^2D^2f_0} \tag{1}$$

where τ is the proper time of the test particle and $D = \int ds = \sqrt{(r^2+b^2)/f_0}$ is the proper distance of the test particle at the equator from the galactic center (we set $a = 0$, and $r_0 = 1$). Observe that for a circular trajectory it follows

$$\frac{d\phi}{dt} = f_0c = c^2\frac{B}{A} \tag{2}$$

Moreover $B = f_0cD$ along the whole galaxy. The first of equations (1) is the second Newton's law for particles travelling onto the scalar field background. We can interpret

$$\frac{d^2D}{d\tau^2} = D\left(\frac{d\phi}{d\tau}\right)^2 - f_0^2c^2D\left(\frac{dt}{d\tau}\right)^2 = \frac{B^2}{D^3f_0^2} - \frac{A^2}{c^2D^3} = \frac{c^2}{D} - \frac{A^2}{c^2D^3} \tag{3}$$

as the force due to the scalar field background, i.e. $F_\Phi = c^2/D - A^2/\left(c^2D^3\right)$. Using the expression for A given in equation (16) of [1], we can write this force in terms of v

$$F_\Phi = -\frac{v^2}{D\left(f_0^2D^2 - v^2/c^2\right)}$$

which corresponding potential is

$$V_\Phi = \frac{1}{2}c^2\ln\left(f_0^2 - \frac{1}{D^2}\frac{v^2}{c^2}\right) \tag{4}$$

Of course, as the author of the comment pointed out, this potential corresponds to non stable trayectories. Nevertheless, $v^2/c^2 \simeq 10^{-6}$, and $f_0 \simeq 0.01\ kpc^{-1}$, this means that potential V_Φ is almost constant for $D \gtrsim 0.3\ kpc$, which corresponds to the region where the solution is valid. At the other hand, we know that the luminous matter is completely Newtonian. The Newtonian force due to the luminous matter is given by $F_L = GM(D)/D^2 = v_L^2/D = B_L^2/D^3$, where v_L is the circular velocity of the test particle due to the contribution of the luminous matter, given by equation (23) in [1] and $B_L = B_L(D)$ is its corresponding angular momentum per unit of mass, given by equation (24) in [1]. The total force acting on the test particle is then $F = F_L + F_\Phi$. For circular trajectories $d^2D/d\tau^2 = F = 0$, then

$$F = \frac{B_L^2}{D^3} - \frac{A^2}{c^2D^3} + \frac{c^2}{D} = 0 \tag{5}$$

which is just equation (18) of reference [1]. The corresponding potential is then $V = V_L + V_\Phi$, but for the regions where $D \gtrsim 0.3\ kpc$, potential V is dominated by the behavior of V_L, which contains stable circular trayectories, like in a galaxy. Therefore the argument of the author of the comment is false, he is not considering the luminous matter in his analysis. Of course, without luminous matter (without a galaxy) the system is unstable. For regions with $D \lesssim 0.3\ kpc$ the solution in not valid any more due to the approximations we have carried out.

Therefore the last conclusion of the comment is completely wrong, the hypothesis of the scalar dark matter is well justified at galactic level (see also [5]) and at the cosmological level too [3, 6, 7]. But at this moment it is only that, a hypothesis which is worth to be investigated.

References

[1] F. S. Guzmán and T. Matos, *Class. Quantum Grav.* **17** (2000) L9

[2] T. Matos, D. Núñez, F. S. Guzmán and E. Ramírez. "Conditions on the Flat Behavior of the Rotational Curves in Galaxies", to be published. Preprint astro-ph/0005528

[3] T. Matos and L. A. Ureña-López. "Quintessence and Scalar Dark Matter in the Universe", *Class. Quantum Grav.* **17** (2000), in press. Preprint astro-ph/0004332

[4] T. Matos and F. S. Guzmán, *Ann. Phys. (Leipzig)* **9** (2000) S133. Available at astro-ph/0002126

[5] T. Matos, F. S. Guzmán and D. Núñez, "Spherical Scalar Field Halo in Galaxies". Preprint astro-ph/0003398

[6] P. J. E. Peebles, Preprint astro-ph/0002495

[7] V. Sahni and L. Wang, Preprint astro-ph/9910097